Reply to the comment on "Sommerfeld effect as the vertex correction in three-dimensional space"

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April 12, 2019

Dear Referee,

We appreciate you taking the time to review our paper [1]. We now communicate with you by responding to your comment on the paper.

From the review report, we learn that you criticized the concept of the reciprocal wave function (and therefore the representation transform for it, e.g., the Eq.(15)). Since the crucial physics behind the Eq.(15) is the concept of the reciprocal wave function[1], for the two comments, we might respond to your criticism on this point first.

Comment:
The author mentions in the abstract that they introduce the reciprocal wave-function and motivate its physical meaning in Appendix A. What is argued in this appendix is that the square modulus of the ratio of the reciprocal wave-functions at two different points q1 and q2 determines the relative probability of *not* finding the system at q1 versus at q2. I strongly disagrees with this statement, as this quantity should instead be $(1-|phi(q1)|^2dq)/(1-|phi(q2)|^2dq)$. This quantity cannot be computed solely from the ratio of the reciprocal wave-functions; thus, the author's statement is incorrect.

Response 1:
As noted in the appendix A[2] and also the mian text (above the Eq. (15)), we are dealing with the wave functions of continuous spectrum. Since the Schrödinger wave function ψ is not normalizable, differing from the case of discrete spectrum, the absolute square of the wave function $|\psi(q)|^2$ is not meaningful in an absolute sense, i.e., we **cannot** have $\int|\psi(q)|^2dq=1$ by choosing a "normalization constant" for the wave function, and the quantities like $|\psi(q)|^2dq$ are *meaningless* in isolation. Thus, for continuous spectrum, wave functions can only determine the *relative* probabilities between different points [2]. The

[1] The representation transform is just an inevitable corollary of the existence of the reciprocal wave function (or the state ket) which can describe a state.

[2] See, e.g., "we recall that in quantum mechanics the meaning of the absolute probability amplitude for the normalized wave function is changed to the relative probability amplitude for the continuous spectral wave function" (remark 1 at the beginning of Appendix A.)

only meaningful quantity here would be the ratio between two points, i.e., $|\psi(q_1)|^2/|\psi(q_2)|^2$. Then the wave function is called *relative probability amplitude* [3], not probability amplitude.

Hence, we see that the "not finding probability" expression "$1-|\psi(q_1)|^2 dq$" is meaningless. We do *not* have the 1, and $|\psi(q_1)|^2 dq$ does *not* stand for the (real) probability of finding the particle in the small interval.

[We may note again that our "relative nonfinding probability", just like the relative finding probability, is not meaningful in an absolute sense. Both of them are meaningful in a relative sense[3].]

Comment:

In equation 15, the author asserts that the "representation transform" (in the sense of a generalization of the Fourier transform to non-integrable functions) of a function 1/phi is the reciprocal of the representation transform of phi. In short, calling the representation transform F, they claim that F[1/phi] = 1/F[phi]. The author fails to provide references proving this result and defining the representation transform. Moreover, this result looks highly dubious to me. Note for example that the Fourier transform of the principal value distribution is by no means the reciprocal of the Fourier transform of the identity function. Since equation 15 is used in the rest of the calculation to compute the "Fourier transform" of a ratio of Green's function, the validity of this step is crucial to the validity of the whole paper.

Response 2:

Essentially, as noted in the first footnote, this comment is still about the existence of the reciprocal wave function.

[We underline by the appendix A that the transform for the reciprocal form should be understood as a representation transform. The representation transform allows us to transform nonintegrable functions into a different representation, and the transform, like many procedures in physics, could be understood as a formal procedure[4].]

Unfortunately, we cannot *prove* the Eq. (15), since the physics behind it, i.e., the nonfinding probability interpretation for the reciprocal wave function, like the finding probability interpretation for the wave function, *cannot* be proven. They are closely interrelated. If we can prove the nonfinding probability interpretation, in principle we can also prove the Born probability interpretation, and that is impossible[5]. Instead, in this work we use three fully independent ways to justify the representation transform for the reciprocal form. One is the formal argument in the main text, another is the physical interpretation and discussion given by Appendix A, and the third is obtaining the desired result

[3]It is important to notice that the final physical result in our work also comes from the ratio between two quantities (the Eqs.(33) and (34)), and neither of the quantities is meaningful in an isolated situation.

[4]Recall that for the quantum physics of continuous spectrum, quantum field theory say, the mathematical foundation is really poor, if not nonexistent. However, our understanding for the laws of Nature proceeds.

[5]For the existing knowledge of physics.

for an independently-originated problem by employing the transform[6].

We hope you may find the response is useful for considering whether our paper deserves a reconsideration for a publication.

Thanks for your time.

Best regards,
Zhentao Zhang

References

[1] Z. Zhang. Sommerfeld effect as the vertex correction in three-dimensional space, arXiv:1701.06856.

[2] See, e.g., L. D. Landau and E. M. Lifshitz. *Quantum mechanics: non-relativistic theory*, Pergamon Press, 1977, p. 7.

[3] See, for example, P. A. M. Dirac. *The principles of quantum mechanics*, Oxford university press, 1958, p. 73.

[6] See the third footnote. Employing the Eq. (15) itself does not give a meaningful result in isolation.