Equivalent paths for describing the physical reality

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The reciprocal wave function was introduced into quantum mechanics, and it was asserted that this wave function describes the physical reality. In this note we shall show, from the more general standpoint, why this novel description of the physical reality exists.

The origin of wave mechanics is the Schrödinger wave function, and one of the postulates in quantum physics is Born's probability interpretation of the wave function [1]. In the previous paper [2], a new but related wave function was introduced into quantum mechanics, which reads

$$\bar{\psi}(q) = \frac{1}{\psi(q)}, \tag{1}$$

where $\psi(q)$ is the Schrödinger wave function in the configuration space.

It was asserted that

$$\bar{\psi} = \psi \tag{2}$$

in physics, i.e., the reciprocal wave function $\bar{\psi}$ represents the same physical reality as the wave function ψ does, and that it is *the relative non-finding probability amplitude* [2]. Then there is the representation transform for the reciprocal forms of the wave function and the other derivative functions in the consistent scheme. As a consequence in practical application [2], we are able to calculate the vertex correction in quantum electrodynamics in the nonperturbative region through an unordinary renormalization approach, and also show the well-known phenomenologically-defined Sommerfeld factor [3] is the vertex correction.

However, due to the interpretation and the formulation for the reciprocal wave function are counterintuitive, the novel description of the physical reality might not be readily understood, though we have carefully explained the assertion in the preceding work. I am convinced that it will be valuable to further explain the underlying idea behind this novel description. Hence, this note is devoted to showing, from a more general standpoint, *why* the reciprocal wave function is the relative non-finding probability amplitude that also describes the physical reality.

I shall now note a simple similarity in elementary arithmetic as my starting point.

A similarity between minus and reciprocal operations

Literally, we all know that for arbitrary positive numbers a and b,

$$a - b = a + (-b). \tag{3}$$

And there is

$$a - b = -[(-a) + b]. \tag{4}$$

The minus $-a$ may be understood as the minus operation $-\mathbf{1}$ is applied to a, where $\mathbf{1}$ is the identity map.

Similarly, using the reciprocal operation $\mathcal{D}$ [2], we may write

$$a \div b = a \times (\mathcal{D}b), \tag{5}$$

where $\mathcal{D}b = \frac{1}{b}$ and $b \neq 0$.

Since the double operation $\mathcal{D}^2 = \mathbf{1}$, we have

$$a \div b = (\mathcal{D}^2 a) \times (\mathcal{D}b) = \mathcal{D}[(\mathcal{D}a) \times b], \tag{6}$$

where $a, b \neq 0$.

We can see the reciprocal operation plays a very similar role in multiplication and division as the minus operation does in addition and subtraction.

The relative information

We now apply a simple classification to informations. The information may be called *absolute information*, if the description for a situation is expressed without any intention of direct comparison. The information may be called *relative information*, if the description is expressed in a relative sense. The meaning of "information" used here is a description that reflects a situation.

The concept of absolute information is not interesting, and we shall concentrate on the concept of relative information. Clearly, there are two basic kinds of relative informations in mathematics: one is defined by subtraction; the other is defined by division.

After noticing the similarity between minus and reciprocal operations in elementary arithmetic, for the two categories of relative informations, we are naturally interested to consider the similar roles the operations of minus and reciprocal might play in the formulations.

In fact we may observe that *the mathematical consequence of applying the reciprocal operation to the numerical result from the relevant comparison can be, logically,*

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equivalent to changing the original order of the two objects in the comparison, and the similar fact also occurs for the minus operation. The abstract change for the comparison, of course, has no effect on the reality, and however it *does* have an effect on the formulation for the reality. In an abstract sense, for the formulations the minus and the reciprocal operations can turn the original relative informations into the informations in the "opposite form". Here the opposite form means the numerical result for a comparison actually comes from the comparison in the opposite order.

It is expected that sometimes the reality described in the opposite form may be "awkward" for us, just like the relative information that describes a thing in the "normal form". However, awkward or not is subjective, and the existence of an equivalent description for a relative information is objective. Since the informations in the two forms reflect the same reality, they may be called *identical informations.*

Next, for our purpose, we shall focus on the concept of the relative probability which is obviously a kind of relative information.

The relative probabilities

We know that the Schrödinger wave function is postulated to be the probability amplitude of finding for the bound state, and the relative finding probability amplitude for the scattering or free state.

a. Scattering state

For the continuous spectrum, the finding probability amplitude cannot have a meaning in the absolute sense. Thus, logically the probability descriptions of finding and not finding are equal, and it is easily understood that the wave function and the reciprocal wave function describes the same physical reality [2]. Therefore, the reciprocal wave function may actually be understandable for the scattering state.

Nevertheless, we may give a note about the free state. In general, the reciprocal of the wave function $\psi(\mathbf{r})$ would not satisfy the Schrödinger equation. However, the plane wave is kind of exceptive, since the reciprocal and the complex conjugate are the same from the mathematical viewpoint for $\psi_0(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}$. Then the abstract quantity $e^{-i\mathbf{k}\cdot\mathbf{r}}$ can have an independent physical meaning, i.e., we may define that it represents the free particle carrying the momentum $-\mathbf{k}$. So, it may be useful to notice that the reciprocal wave function $\bar{\psi}_0(\mathbf{r})$ represents the different physical reality.

Realize the absolute squares of the wave function and the reciprocal wave function are exchangeable under the reciprocal operation. We may remark that, from the more general standpoint, *the relative non-finding probability description may be viewed as a logical interpretation for the description of the relative finding probability in the opposite form, due to the fact that the finding and the nonfinding are contrary to each other.*

b. Bound state

For the wave function of the discrete spectrum, we know that it is square integrable, and after the normalization the finding probability can have a meaning in the absolute sense. Consequently, there is also the absolute non-finding probability for the bound state, which is clearly not formulated by the reciprocal wave function. Then it seems that the concept of the reciprocal wave function would not make any sense for the bound state. But, my point is that logically the relative non-finding probability is formulated as a universal concept, and that it should not be only valid for the scattering state. Thus, let us now consider the passive description of the relative probability for the bound state.

From the *purely practical* viewpoint, indeed, the wave function and the reciprocal wave function may not be equal in logic, since the relative non-finding probability cannot have an absolute meaning for seeking the position of the particle in the configuration space. However, we may recall a basic fact in quantum mechanics: the relative finding probability is more important than the absolute finding probability for our abstract description of the physical reality. This fact is easily appreciated, since ψ and $c\psi$ represent the same physical reality, where c is any, not zero, constant. And notice that the procedure of normalization is only relevant to the concept of the probability-related measurement. Thus, from the point of view of *the abstract description of the physical reality*, the situation of the bound state is not essentially different from the scattering state.

However, to further make our point clear, we shall show a failure for the concept of the absolute non-finding probability in quantum physics.

Let the normalized wave function $\psi(\mathbf{r})$ represents a state of a bounded particle. The classical non-finding probability $P_{\mathrm{non}}(\mathbf{r})$ for a "point", an element of volume, is

$$P_{\mathrm{non}}(\mathbf{r}) = 1 - |\psi(\mathbf{r})|^2 d^3r. \tag{7}$$

It may be remarked here that differing from the concept of the probability for finding the bounded particle at a point, this absolute non-finding probability, in fact, is not a "logically-isolated" information merely regarding that point.

The probability of detecting the bounded particle in the volume element d^3r cannot be a finite value. Thus, the ratio of the classical non-finding probabilities at two points

$$R_{\mathrm{non}}(\mathbf{r_1}, \mathbf{r_2}) = \frac{P_{\mathrm{non}}(\mathbf{r_1})}{P_{\mathrm{non}}(\mathbf{r_2})} = 1. \tag{8}$$

This formula tells us that the possibilities of not finding the particle at different points are the same. One cannot deny this result is perfectly correct, especially for considering the real points, but we should admit *this absolutely-compared formulation cannot adequately characterize the situation from the viewpoint of nonfinding*. Besides, it *loses entirely the relative information formulated by the finding probability amplitude at two points*, i.e., the probability ratio $R[=|\psi(\mathbf{r_i})|^2/|\psi(\mathbf{r_j})|^2]$, which is the information that really matters for the probability description of the physical reality.

Although it would not be the root cause for this failure, it is still instructive to notice the fact that the relative informations defined by subtraction and division are *not* identical informations and that the absolutely-compared formulation mixed subtraction and division.

For the relative non-finding amplitude, one may understand that it is introduced to define *a type of logically-isolated information that can simply concern a point from the non-finding viewpoint*. Hence, there is an *equal-right relation* that

$$\bar{R}(\mathbf{r_1},\mathbf{r_2})=\bar{R}(\mathbf{r_3},\mathbf{r_4}), \tag{9}$$

if $R(\mathbf{r_1},\mathbf{r_2})=R(\mathbf{r_3},\mathbf{r_4})$, and conversely, where $\bar{R}$ is the probability ratio formulated by the relative non-finding probability amplitude.

Perhaps a note is helpful. The bound state wave function goes to zero at infinity, and then the reciprocal wave function would be divergent at infinity. However, we should always keep in mind that the relative non-finding probability amplitude is not meaningful in the absolute sense. Thus it is not necessary to worry about the possible divergences at some points. As a matter of fact, the value of the relative non-finding probability at a point almost always diverges, and that does not matter.

At the end of this section, it may be interesting to point out a "dual correspondence" for the active and passive probability descriptions: the total (relative) non-finding probability may have a formal meaning in the absolute sense for the scattering state, since $1/\int|\psi|^2dq=0$.

Remark

The concept of the relative non-finding probability and the ramifications are not derived from classical concepts. However, it is shown that they are logical results after careful consideration. And this may achieve the aim of this work.

[1] M. Born, Z. Phys. **37**, 863 (1926).

[2] Z. Zhang, arXiv:1701.06856.

[3] A. Sommerfeld, Annalen der Physik **403**, 257 (1931).